Advanced Course on FAIRSHAPE

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1 PREAMBLE

Assembled here is a collection of articles presented at the *Advanced Course on "Automatic Fairing and Shape-Preserving Methodologies for CAD/CAM"* held at Pfalz Akademie, Lambrecht (near Kaiserslautern), Germany, during the period of March 25th to 29th, 1996. The course has been organized by the Network **FAIRSHAPE**, a two-year EU-project, which is economically supported by the *XII* Directorate General of the European Commission in the context of the Framework Programme: **H**uman **C**apital and **M**obility (**HCM**, 1992-1994).

The objectives of FAIRSHAPE can be classified into two major categories, namely the *general* objectives and the *technical* ones. The general objectives, which are of strategic character, can be summarized as follows:

1. Initiate and establish close interaction between European Organizations, which are involved in the CAGD/CAD/CAM activity by doing either basic or applied research or by exploiting the results of applied research.
2. Provide the academic partners of the network with CAGD problems, which are of current industrial interest.
3. Reduce the time needed for the results of basic research in CAGD to be disseminated to the CAD/CAM end-users and tested in the context of a real CAD/CAM environment.
4. Exchange methods and expertise between partners belonging to different industrial sectors, more specifically, the automobile and marine sector.

The major technical goals of FAIRSHAPE are the following:

1. Create a knowledge base containing the state-of-the-art on *fairing* and *shape-preserving* methods for curves and surfaces.
2. Compare the methods, classified in (1), from a theoretical point of view and evaluate the best of them by means of benchmark tests prepared by the industrial partners of FAIRSHAPE.
3. Based on the results obtained in (2), develop new automatic algorithms for constructing fair or shape-preserving interpolating curves/surfaces.
4. Formulate and solve practical problems, in which the designer needs to perform fairing and shape-preservation, simultaneously.
5. Establish criteria for shape-quality control and derive computationally efficient implementations of them. Check the so-obtained criteria in real CAD/CAM environment prepared by the industrial partners of FAIRSHAPE.

Besides *the Exchange and Training of Young Researchers*, which is the principal activity of the network, the Steering Committee of FAIRSHAPE is also organizing *Internal Workshops*, for monitoring the materialization of the afore-mentioned strategic and technical goals. So far, we have organized two internal workshops, the first in Athens (March '95) and the second in Berlin (August '95). Furthermore, and in accordance with the Work Programme adopted by the European Commission, the activities of FAIRSHAPE should also include the organization of

1. An **Advanced Course** and
2. A *Benchmarking Workshop.*

The purpose of the Lambrecht Advanced Course was two-fold. First, provide an, as complete as possible, description of the current status of the basic research done in the areas of *fairing* and *shape-preserving interpolation.* Secondly, report and submit to the criticism of a broader audience (46 participants) the so-far obtained results of the bench-marking activities of FAIRSHAPE in the areas of *fairing*, *shape-preserving interpolation* and *constrained approximation.* We believe that both purposes have been fully materialized and hope that this is reflected in the diversity and quality of the papers contained in the present volume. On this occasion, I would like to express, also on the behalf of my colleagues in FAIRSHAPE, my thanks to the members of the Differential-Geometry-and-Kinematics-Group of the Technical University of Darmstadt, and especially Professor J. Hoschek, for their contribution in organizing the whole event at Lambrecht and editing the articles of the collection in hand.

Let me conclude this preamble by making a reference to the internal structure of our network. FAIRSHAPE comprises partners from both academia and industry. More specifically, the partners of FAIRSHAPE can be classified into three major groups:

1. *The academic group, which is involved in basic research in the area of CAGD.* This group consists of:

(a) Differential Geometry and Kinematics Group, Department of Mathematics, Technische Hochschule Darmstadt (DE), *senior scientist: Prof. J. Hoschek*,

(b) Numerical Analysis Section, Department of Energetics, University of FLorence (IT), *senior scientists: Prof. F. Fontanella and Prof. P. Costantini*,

(c) Department of Applied Mathematics, University of Zaragoza (SP), *senior scientists: Prof. M. Gasca and Prof. J. Carnicer*,

(d) Department of Applied Mathematical Sciences, School of Mathematics, University of Leeds (UK), *senior scientists: Prof. M. Bloor and Prof. M. Wilson*, and

(e) Department of Mathematical Sciences, The University, Dundee, *senior scientist: Prof. T.N.T. Goodman.*

2. *The academic group, which is involved in applied research in the areas of CAGD and CAD/CAM.* This group consists of:

(a) Division of Ship Design, Department of Naval Architecture & Marine Engineering, Technische Universität Berlin (DE), *senior scientist: Prof. H. Nowacki*, and

(b) Ship-Design Laboratory, Department of Naval Department of Naval Architecture & Marine Engineering, National Technical University of Athens (GR), *senior scientist: Assoc. Prof. P.D. Kaklis.*

3. *The industrial group*, consisting of:

(a) The SYRKO-CAD/CAM Development Group, Mercedes-Benz AG, *senior scientist: Dr. E. Kaufmann*, and

(b) KCS (Kockums Computer Systems) UK, Ltd., *senior scientists: D. Catley and A. Ives-Smith.*

P.D. Kaklis,
Project-coordinator.

Contents

Fairing and Shape Preserving of Curves

Experiences in Curve Fairing

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Abstract: The task of fairing a curve interpolating a given point data set and potentially given end conditions by minimizing an explicit fairness measure is discussed from the viewpoint of the resulting curve quality. Results are compared for different choices of a fairness criteria applied to a variety of data sets. The improvements achievable by going from integer to rational cubic B–spline curves are examined in particular. Fairness quality can be raised both by lessening the constraints and by increasing the freedoms in curve representation.

1 Introduction

One of the objectives in the FAIRSHAPE project (HCM-Network CHRX-CT94-0522) is the fairing of curves with given interpolation constraints. The main interest is in comparing curve shapes when different fairness criteria and varying curve representations are applied. The objective is to improve the fairness quality of curves as measured by some explicit fairness criterion.

The special case of cubic spline interpolation of a given point set is a well-known classical problem of curve fitting. It is well understood that this interpolation problem is uniquely defined except for two missing conditions, e.g., end conditions [1]. The missing conditions can be replaced by optimality requirements, e.g., by minimizing a fairness measure [2]. This has been done in practice, however the cubic spline interpolation problem is highly constrained by the point data set and the quality of curve thus is usually not very sensitive to the fairing process.

A contrasting situation exists when a sufficient surplus of freedoms is introduced into the curve representation, e.g. by raising the polynomial degrees, increasing the number of segments etc., so that the problem statement becomes more strongly underdeterminate. In this case the shape of the curve becomes more responsive to the fairing objectives. Meier [3] and Meier, Nowacki [4] have drawn the attention to this option and have documented the results of shape improvements by a curve fairing process with interpolation.

The current investigation resumes the basic premise of this earlier work. The working hypothesis is that an improved fairness quality can be achieved by introducing additional degrees of freedom in the curve representation. This will be illustrated by several examples.

In principle the curve representation may be extended to offer additional flexibility by:

- Raising the polynomial degrees [3],
- Increasing the number of curve segments,
- Changing the parametrization of the curve knot vector, especially from uniform to non-uniform,
- Moving from integer to rational representations, thus bringing in free weighting factors, e.g., by going from NUBS to NURBS curves.

Although several of these options were explored in our current work, we will mainly report results from our comparisons between NUBS and NURBS. They are indicative of the potential of shape improvement in such fairing processes.

A second main interest was in the influence of the choice of fairness measure upon curve shape, a main theme of the FAIRSHAPE project. A variety of fairness criteria was applied to several data sets and the effects on fairness measures and curve shape were recorded.

The results reported here are part of an ongoing activity whose goal it is to find a proper balance between data fidelity and shape quality in fairing process.

2 Fairness Criteria

In the current context the fairness of a curve is considered to be a global property of the curve that can be measured and evaluated by some suitable integral criterion taken over the curve range. The choice of the criterion is somewhat subjective and should depend on the purpose of the fairing process. In fact, in some applications the criterion may also be a functional, non-geometric integral property of the shape, like the viscous drag of a foil.

Whatever the choice of fairness criterion may be, it is intended to reflect the design objective and does result in a single measure of merit, a "fairness number", as a measure of shape quality.

In the current paper we have considered the following fairness criteria:

$$E_1 = \int_0^1 |\dot{\overline{Q}}(u)|^2 du = \text{first order parametric criterion}$$

$$E_2 = \int_0^1 |\ddot{\overline{Q}}(u)|^2 du = \text{second order parametric criterion}$$

$$E_3 = \int_0^1 |\dddot{\overline{Q}}(u)|^2 du = \text{third order parametric criterion}$$

$$E_k = \int_0^1 k^2(s) ds = \text{curvature criterion}$$

$$E_{k'} = \int_0^1 |dk(s)/ds|^2 ds = \text{change of curvature criterion}$$

where

$\overline{Q}(u)$	= parametric curve
$\dot{\overline{Q}}(u), \ddot{\overline{Q}}(u), \dddot{\overline{Q}}(u)$	= First, second, third parametric derivatives of $\overline{Q}(u)$
$k(s)$	= curvature as a function of arc length s

3 Fairing Process

A fairing process has a data set and an assumed curve representation as inputs and is defined by a fairness criterion and constraints (Fig. 1).

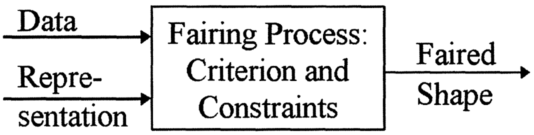

Fig. 1 Definition of Fairing Process

We use the following **test data sets** in this report:

- Meier's test data points [3]
- A tanker afterbody section
- A historical frigate midship section

All test examples are for planar curves.

The **curve representations** are:

- Non-uniform integer and rational B–splines (NUBS and NURBS)
- with a polynomial degree of 3 (potentially higher)
- and with variable segmentation

The **fairness criteria** are those recorded in Section 2.

Constraints consist of free or fixed end conditions (with or without end tangent vectors).

A workbench software environment was created at TU Berlin in order to do systematic variations on this set of assumptions in the statement of the fairing process. Existing software components were integrated into the workbench (Fig. 2). The point data are initially interpolated to generate a NUBS curve representation. This can readily be rewritten as a NURBS curve with unit weights, which forms the input to the optimization process. The result of the fairing process is a NURBS curve with optimized weights and its associated fairness criteria. The results are displayed by the visualization component of the workbench.

The idea of fairing NURBS curves by optimizing on the weights was first proposed by Hohenberger and Reuding [5], who for approximation of point data used a global change of curvature criterion as their fairness measure. In the current context the approach is extended to interpolation and any desired fairness criterion.

The optimization process of the weights is here implemented in a simplified way to save computer time. The weights w_i associated with the control points $\overline{P}_i$ of the defining polygon of the NURBS curve are constrained to be varied according to a cubic Bézier curve relationship

$$w_i = k_0 B_0\left(\frac{i}{n}\right) + k_1 B_1\left(\frac{i}{n}\right) + k_2 B_2\left(\frac{i}{n}\right) + k_3 B_3\left(\frac{i}{n}\right)$$

where
$B_j(u)$ = cubic Bernstein basis functions, j = 0, ... ,3
and
k_0, k_1, k_2, k_3 = free optimization parameters in curve range

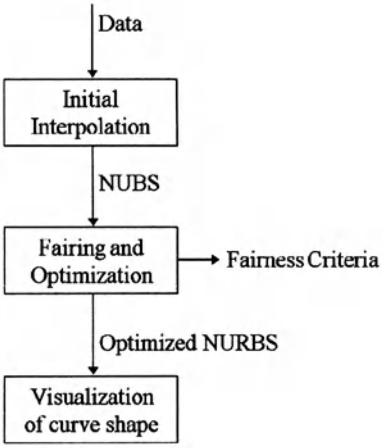

Fig. 2 Fairing Process in Workbench

This limits the number of free parameters to four per curve even when the curve has many control points.

The numerical optimization algorithm used here to minimize the fairness functional is an unconstrained minimization technique, viz., the well known simplex method of downhill search by Nelder and Mead [6]. It converges rather reliably in reasonable computing time.

4 Results

Figs. 3 to 7 show several results of the NURBS fairing tests. The curves are represented by piecewise cubic NURBS with non-uniform chord length parametrization and tangential end conditions throughout these examples. The initial curve with unit weights (NURBS curve) was optimized by varying the weights. The fairness criterion was systematically varied including E_1, E_2, E_3 and E_k. The optimization succeeded in all cases to reduce the fairness criterion compared to the initial status although in some cases only slightly.

Fig. 3 shows an interpolating cubic NURBS curve for Meier's data set with given end tangent vectors and chord length parametrization. The curvature criterion E_k is minimized. The faired curve differs in the middle segments and exhibits reduced curvature (flatness). This is typical for curvature or strain energy minimization. The criterion was reduced by about 13 percent.

Fig. 4 presents the HSVA tanker cross section with 8 non-equidistant data points and end tangent constraints. The E_1 criterion is applied. It reflects the intention to reduce arc length and does result in greater tautness of the curve, although numerically it changes only slightly. Note that the (non-optimal) E_2 decreases while E_k rises. Hence the curvature and the second parametric derivative criteria do not necessarily show equivalent trends.

Figs. 5 to 6 give corresponding results for the E_2, E_3 and E_k criteria. In all cases there are certain improvements in the optimized measure of merit whereas the others may or may not be favourably affected. Table 1 summarizes these results. It shows what penalty must be paid in other criteria when one of the criteria is optimized.

Calculated	*Optimized Criterion*			
Criterion	E_1	E_2	E_3	E_k
E_1	204.0	205.0	205.0	206.1
E_2	11413.	4139.	6216.	53557.
E_3	$1.25 \cdot 10^8$	$0.92 \cdot 10^8$	$0.75 \cdot 10^8$	$14.6 \cdot 10^8$
E_k	0.980	0.945	0.929	0.924

Tab. 1 Fairness Criteria for HSVA Tanker Section

Although the choice of criterion remains subjective, there are many applications where the E_3 or E_k fairness measure may be favoured because they produce the most gentle changes in curvature. The E_k criterion is still under investigation.

Finally in Fig. 7 the midship section curve of a historical frigate is faired by the curvature criterion which is improved by only 2 percent.

Yet the spike visualization of radii of curvature demonstrates a certain curvature integral reduction in this example.

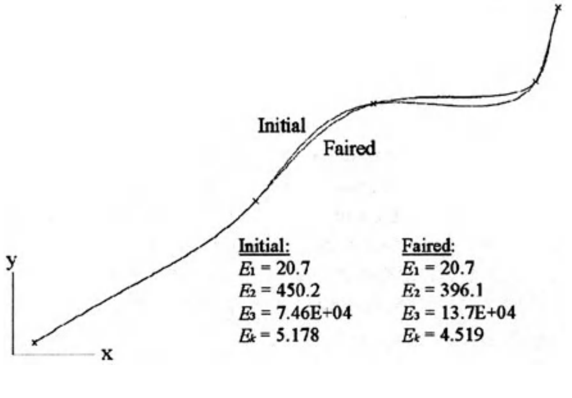

Fig. 3 Initial and faired curve for Meier's test data set.
The E_k criterion is minimized.

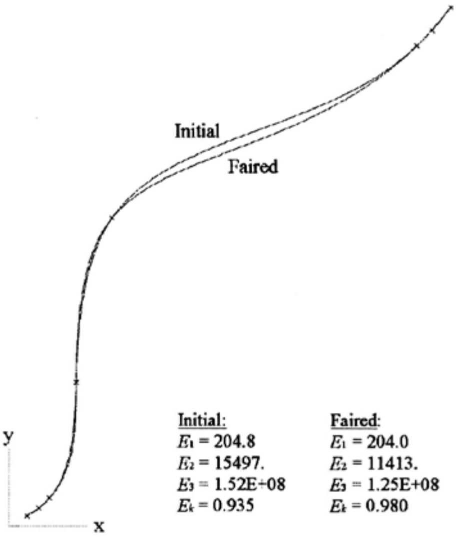

Fig. 4 Initial and faired curve for HSVA Tanker (aft section) data set. The E_1 criterion is minimized.

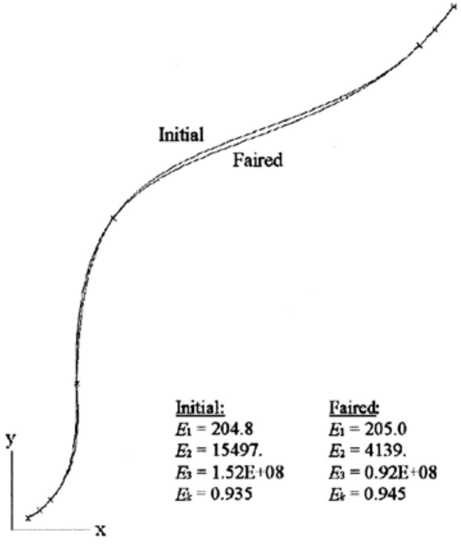

Fig. 5 Initial and faired curve for HSVA Tanker (aft section) data set. The E_2 criterion is minimized.

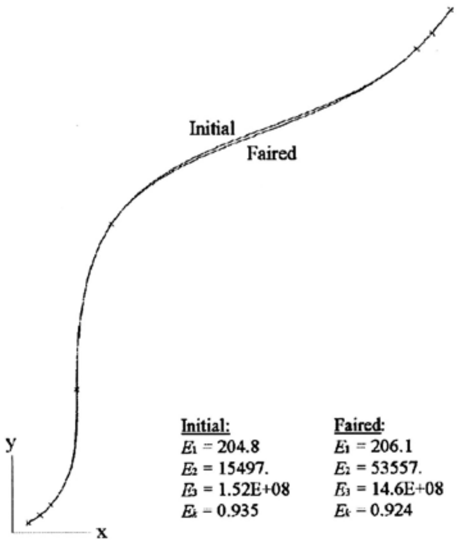

Fig. 6 Initial and faired curve for HSVA Tanker (aft section) data set.
The E_k criterion is minimized.

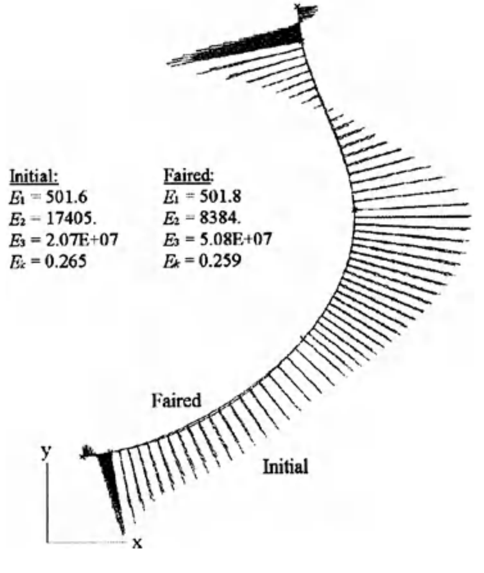

Fig. 7 Initial and faired curve for the historical frigate (midship section) data set.
The E_k criterion is minimized.